

# PARTIAL FRACTION (5 MARK)

10 MARKS
PARTIAL (5)
BINOMIAL (5)

10 MARKS
PARTIAL (5)
INTEGRATION (5)

## D-LEVELS

$$\frac{3}{x+1} + \frac{5}{x+5} \quad \leftarrow \text{ANSWER}$$
$$\frac{3(x+5) + 5(x+1)}{(x+1)(x+5)}$$
$$\frac{3x + 15 + 5x + 5}{(x+1)(x+5)}$$
$$\frac{8x + 20}{(x+1)(x+5)} \quad \rightarrow \text{QUESTION}$$

PARTIAL FRACTION

## FORMS

### LINEAR

power of  $x$  is 1.

$$\frac{3}{(x+1)(x+2)} \equiv \frac{A}{x+1} + \frac{B}{x+2}$$

### REPEATED

one bracket with whole squared power.

$$\frac{3}{(x+5)(x-3)^2} \equiv \frac{A}{x+5} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$$

Memorize this case as variation of REPEATED

$$\frac{3}{x^2(2x+1)} \equiv \frac{A}{x} + \frac{B}{x^2} + \frac{C}{2x+1}$$

QUADRATIC

↓  
x<sup>2</sup> term inside the bracket

$$\frac{3}{(x+5)(x^2+1)} \equiv \frac{A}{x+5} + \frac{Bx+C}{x^2+1}$$

PARTIAL FRACTION

PROPER  
N < D

check max power of x in expanded form in both the numerator and denominator

IMPROPER  
N ≥ D

$$\frac{3x^2+5x+1}{(x+1)(x^2+5)} \quad \frac{2}{3}$$

↓  
x<sup>3</sup>

$$\frac{2x^3-7x^2+1}{(x+1)(x+2)} \quad \frac{3}{2}$$

$$\frac{x^2+3}{(x+1)(x+2)^2} \quad \frac{2}{3}$$

$$\frac{5x^2+6x+8}{(x+1)(x+2)} \quad \frac{2}{2}$$

$$\frac{2x+5}{(x+1)(x+2)} \quad \frac{1}{2}$$

FOR PROPER FRACTIONS YOU CAN USE FORMS GIVEN ABOVE DIRECTLY.

FOR IMPROPER YOU CANNOT USE PARTIAL FRACTION FORMS DIRECTLY.

STEP 1: USE LONG DIVISION

To BRING  $Q + \frac{R}{D}$   
proper ←

STEP 2: Apply partial fraction form on the proper fraction only.

Improper

$$\frac{11}{9} \rightarrow \begin{array}{r} \text{Divisor } 9 \overline{) 11} \\ \underline{-9} \\ 2 \end{array} \rightarrow 1 \frac{2}{9}$$

$1 \rightarrow$  Quotient

$2$   
Remainder.

$$1 + \frac{2}{9} \text{ proper}$$
$$Q + \frac{R}{D}$$

IDENTITY: ( $\equiv$ )

- (1) You cannot switch sides of terms.
- (2) you are allowed to put any value of  $x$ .

1 Let  $f(x) = \frac{x^2 + 7x - 6}{(x-1)(x-2)(x+1)}$ .  $\frac{2}{3}$  (proper) (form applied directly)

(i) Express  $f(x)$  in partial fractions.

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$$\frac{x^2 + 7x - 6}{(x-1)(x-2)(x+1)} \equiv \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x+1}$$

$$\frac{x^2 + 7x - 6}{(x-1)(x-2)(x+1)} \equiv \frac{A(x-2)(x+1) + B(x-1)(x+1) + C(x-1)(x-2)}{(x-1)(x-2)(x+1)}$$

$$x^2 + 7x - 6 \equiv A(x-2)(x+1) + B(x-1)(x+1) + C(x-1)(x-2)$$

$x-1=0$ $x=1$ $(1)^2 + 7(1) - 6 = A(1-2)(1+1)$ $2 = A(-1)(2)$ $A = -1$	$x-2=0$ $x=2$ $(2)^2 + 7(2) - 6 = B(2-1)(2+1)$ $12 = B(1)(3)$ $B = 4$	$x+1=0$ $x=-1$ $(-1)^2 + 7(-1) - 6 = C(-1-1)(-1-2)$ $-12 = C(-2)(-3)$ $C = -2$
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$$\frac{x^2 + 7x - 6}{(x-1)(x-2)(x+1)} \equiv \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x+1}$$

$$\frac{-1}{x-1} + \frac{4}{x-2} - \frac{2}{x+1}$$

5 Let  $f(x) = \frac{7x+4}{(2x+1)(x+1)^2}$ .  $\frac{1}{3}$  (proper) (apply form).

(i) Express  $f(x)$  in partial fractions.

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$$\frac{7x+4}{(2x+1)(x+1)^2} \equiv \frac{A}{2x+1} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

$$\frac{7x+4}{(2x+1)(x+1)^2} \equiv \frac{A(x+1)^2 + B(2x+1)(x+1) + C(2x+1)}{(2x+1)(x+1)^2}$$

$$\boxed{7x+4 \equiv A(x+1)^2 + B(2x+1)(x+1) + C(2x+1)}$$

$$2x+1=0$$

$$x = -\frac{1}{2}$$

$$7\left(-\frac{1}{2}\right) + 4 = A\left(-\frac{1}{2} + 1\right)^2$$

$$\frac{1}{2} = A\left(\frac{1}{4}\right)$$

$$A = 2$$

$$x+1=0$$

$$x = -1$$

$$7(-1) + 4 = C(2(-1) + 1)$$

$$-3 = C(-1)$$

$$C = 3$$

$$(x+1)^2 = 0$$

$$x+1 = 0$$

$$x = -1$$

we need a new value each time.

↓  
you are allowed any value of  $x$ .

Now Put  $x = 0$

$$7(0) + 4 = 2(0+1)^2 + B(2(0)+1)(0+1) + 3(2(0)+1)$$

$$4 = 2(1) + B(1)(1) + 3(1)$$

$$4 = 2 + B + 3$$

$$B = -1$$

$$\frac{7x+4}{(2x+1)(x+1)^2} \equiv \frac{A}{2x+1} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

$$\boxed{\frac{2}{2x+1} - \frac{1}{x+1} + \frac{3}{(x+1)^2}}$$

3 (i) Express  $\frac{3x^2+x}{(x+2)(x^2+1)}$  in partial fractions.  $\frac{2}{3}$  (proper) (apply form). [5]

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$$\frac{3x^2+x}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+1} \quad \rightarrow \text{V.I.M.P}$$

$$\frac{3x^2+x}{(x+2)(x^2+1)} = \frac{A(x^2+1) + (Bx+C)(x+2)}{(x+2)(x^2+1)}$$

$$3x^2+x = A(x^2+1) + (Bx+C)(x+2)$$

$$x+2=0$$

$$x=-2$$

$$3(-2)^2 + (-2) = A((-2)^2 + 1)$$

$$10 = A(5)$$

$$A=2$$

$$x^2+1=0$$

$$x^2=-1$$

No solutions

Now you have to put two new values of  $x$ .

First value  $x=0$

Second value  $x=any$ .

$$x=0 \quad 3(0)^2 + 0 = 2(0^2+1) + (B(0)+C)(0+2)$$

$$0 = 2(1) + (C)(2)$$

$$-2 = 2C$$

$$C = -1$$

$$x=1 \quad 3(1)^2 + 1 = 2(1^2+1) + (B(1)+C)(1+2)$$

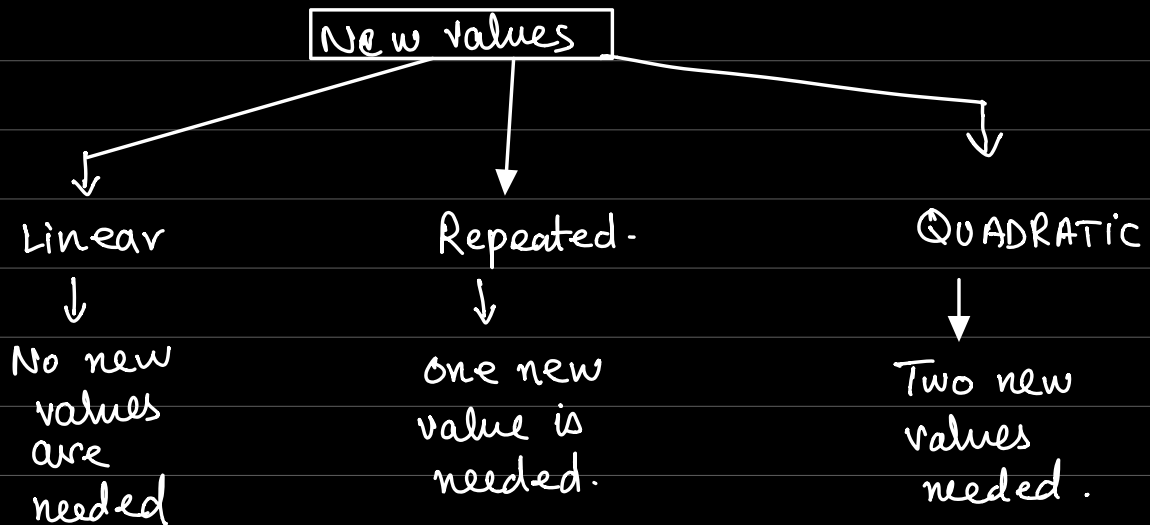
$$4 = 4 + (B-1)(3)$$

$$0 = 3(B-1)$$

$$B=1$$

$$\frac{3x^2+x}{(x+2)(x^2+1)} = \frac{2}{x+2} + \frac{1x-1}{x^2+1}$$

$$= \frac{2}{x+2} + \frac{x-1}{x^2+1}$$



$$x = 0$$

First:  $x = 0$   
 Second:  $x = \text{any}$ .

$$\frac{3x^2 + x}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+1}$$

<sup>3</sup> A      <sup>-2</sup> B      <sup>5</sup> C

$$\frac{3}{x+2} + \frac{-2x+5}{x^2+1} \quad \checkmark$$

$$\frac{3}{x+2} + \frac{-(2x-5)}{x^2+1}$$

$$\frac{3}{x+2} - \frac{2x-5}{x^2+1} \quad \checkmark$$